1 A spherical aluminum ball of mass 1.26 kg contains an empty spherical cavity that is concentric with the ball. The ball just barely floats in water. Calculate (a) the outer radius of the ball and (b) the radius of the cavity

SOLUTION:

(a) The weight of the ball must be equal to the buoyant force of the water:

$$1.26\text{ kg}\,g = \rho_{water}\frac{4}{3}\pi r_{outer}^3 g$$

$$r_{outer} = \left(\frac{3\times 1.26\text{ kg}}{4\pi 1\,000\text{ kg/m}^3}\right)^{1/3} = \boxed{6.70\text{ cm}}$$

(b) The mass of the ball is determined by the density of aluminum:

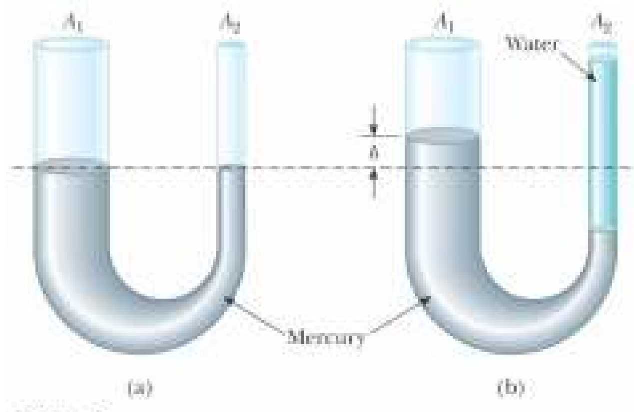
$$m = \rho_{Al}V = \rho_{Al}\left(\frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3\right)$$

$$1.26\text{ kg} = 2\,700\text{ kg/m}^3\left(\frac{4}{3}\pi\right)\left((0.067\text{ m})^3 - r_i^3\right)$$

$$1.11\times 10^{-4}\text{ m}^3 = 3.01\times 10^{-4}\text{ m}^3 - r_i^3$$

$$r_i = \left(1.89\times 10^{-4}\text{ m}^3\right)^{1/3} = \boxed{5.74\text{ cm}}$$

2. Mercury is poured into a U-tube as in Figure P14.18a. The left arm of the tube has cross-sectional area A_1 of 10.0 cm², and the right arm has a cross-sectional area A_2 of 5.00 cm². One hundred grams of water are then poured into the right arm as in Figure P14.18b. (a) Determine the length of the water column in the right arm of the U-tube. (b) Given that the density of mercury is 13.6 g/cm³, what distance h does the mercury rise in the left arm?



(a) Using the definition of density, we have

$$h_w = \frac{m_{water}}{A_2\rho_{water}} = \frac{100\text{ g}}{5.00\text{ cm}^2\left(1.00\text{ g/cm}^3\right)} = \boxed{20.0\text{ cm}}$$

(b) Sketch at the right represents the situation after the water is added. A volume (A_2h_2) of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is A_1h. Since the total volume of mercury has not changed,

$$A_2h_2 = A_1h \qquad \text{or} \qquad h_2 = \frac{A_1}{A_2}h \qquad (1)$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

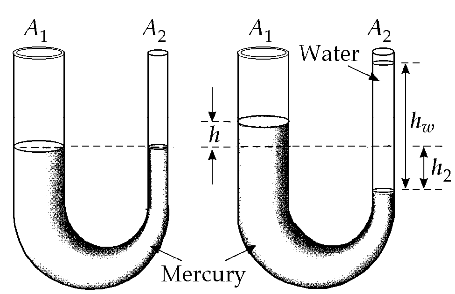
$$P = P_0 + \rho_{water}gh_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{Hg}g(h+h_2) = P_0 + \rho_{water}gh_w$$

which, using equation (1) above, reduces to

$$\rho_{Hg}h\left[1+\frac{A_1}{A_2}\right] = \rho_{water}h_w \quad\text{or}\quad h = \frac{\rho_{water}h_w}{\rho_{Hg}\left(1+\frac{A_1}{A_2}\right)}.$$



(a) (b)

Thus, the level of mercury has risen a distance of

$$h = \frac{\left(1.00\text{ g/cm}^3\right)(20.0\text{ cm})}{\left(13.6\text{ g/cm}^3\right)\left(1+\frac{10.0}{5.00}\right)} = \boxed{0.490\text{ cm}}\ \text{above the original level}$$

3 A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is 2.50×10^{-3} m^3/min, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

1 SOLUTION: Assuming the top is open to the atmosphere, then

2 $P_1 = P_0$. Note $P_2 = P_0$.

Flow rate $= 2.50 \times 10^{-3}\ \text{m}^3/\text{min} = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$.

(a) $A_1 >> A_2$ so $v_1 << v_2$

Assuming $v_1 = 0$,

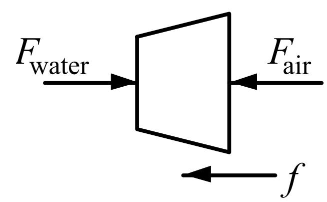
$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

$$v_2 = (2gy_1)^{1/2} = [2(9.80)(16.0)]^{1/2} = \boxed{17.7\ \text{m/s}}$$

(b) Flow rate $= A_2 v_2 = \left(\frac{\pi d^2}{4}\right)(17.7) = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$ so $d = \boxed{1.73 \times 10^{-3}\ \text{m}} = 1.73\ \text{mm}$

4. A village maintains a large tank with an open top, containing water for emergencies. The water can drain from the tank through a hose of diameter 6.60 cm. The hose ends with a nozzle of diameter 2.20 cm. A rubber stopper is inserted into the nozzle. The water level in the tank is kept 7.50 m above the nozzle. (a) Calculate the friction force exerted on the stopper by the nozzle. (b) The stopper is removed. What mass of water flows from the nozzle in 2.00 h? (c) Calculate the gauge pressure of the flowing water in the hose just behind the nozzle.

Take point ① at the free surface of the water in the tank and ② inside the nozzle.



(a) With the cork in place $P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

becomes $P_0 + 1\,000\ \text{kg/m}^3\ 9.8\ \text{m/s}^2\ 7.5\ \text{m} + 0 = P_2 + 0 + 0;$

$P_2 - P_0 = 7.35 \times 10^4\ \text{Pa}$.

For the stopper $\sum F_x = 0$

$F_{\text{water}} - F_{\text{air}} - f = 0$

$P_2 A - P_0 A = f$

$f = 7.35 \times 10^4\ \text{Pa}\,\pi(0.011\ \text{m})^2 = \boxed{27.9\ \text{N}}$

(b) Now Bernoulli's equation gives $P_0 + 7.35 \times 10^4\ \text{Pa} + 0 = P_0 + 0 + \frac{1}{2}(1\,000\ \text{kg/m}^3)v_2^2$

$v_2 = 12.1\ \text{m/s}$

The quantity leaving the nozzle in 2 h is

$$\rho V = \rho A v_2 t = (1\,000\ \text{kg/m}^3)\pi(0.011\ \text{m})^2(12.1\ \text{m/s})7\,200\ \text{s} = \boxed{3.32 \times 10^4\ \text{kg}}.$$

(c) Take point 1 in the wide hose and 2 just outside the nozzle. Continuity:

$A_1 v_1 = A_2 v_2$

$$\pi\left(\frac{6.6\ \text{cm}}{2}\right)^2 v_1 = \pi\left(\frac{2.2\ \text{cm}}{2}\right)^2 12.1\ \text{m/s}$$

$$v_1 = \frac{12.1\ \text{m/s}}{9} = 1.35\ \text{m/s}$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 + 0 + \frac{1}{2}(1\,000\ \text{kg/m}^3)(1.35\ \text{m/s})^2 = P_0 + 0 + \frac{1}{2}(1\,000\ \text{kg/m}^3)(12.1\ \text{m/s})^2$$

$$P_1 - P_0 = 7.35 \times 10^4\ \text{Pa} - 9.07 \times 10^2\ \text{Pa} = \boxed{7.26 \times 10^4\ \text{Pa}}$$